[5]Siler, L.G. and Martindale, W.R., "Test Results from the NASA Space Shuttle Orbiter Heating Test (MH-2) Conducted in the AEDC-VKF Tunnel, B," AEDC-DR-75-103, Oct. 1975.

[6]Siler, L.G., "Test Results from the NASA Space Shuttle Orbiter Heating Test (MH-1) Conducted in the AEDC-VKF Tunnel F," AEDC-DR-76-13, May 1976.

[7]Wannenwetsch, G.D. and Martindale, W.R., "Roughness and Wall Temperature Effects on Boundary-Layer Transition on a 0.0175 Scale Space Shuttle Orbiter Model Tested at Mach Number 8," AEDC-TR-77-19, April 1977.

[8]Bertin, J.J., Idar, E.S., and Goodrich, W.D., "Effect of Surface Cooling and Roughness on Transition for the Shuttle Orbiter," AIAA Paper 77-704, June 1977.

Satellite Attitude Determination by Simultaneous Line-of-Sight Sightings

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SUPPOSE that a satellite carries two onboard angle trackers that can make simultaneous measurements of the lines-of-sight to two known references, e.g., two stars, or one star and one Earth-landmark. Then the vehicle attitude at the instant of the sightings can be determined as a linear combination of unit vectors along the lines of sight and their cross product, where the coefficients are functions of the azimuth and elevation of the angle trackers.

Analysis

Let L_1, L_2 be unit vectors along the known lines-of-sight. These ultimately are to be expressed in components in some inertial reference frame. Suppose that L_1, L_2 also are measured in the vehicle frame in terms of azimuth and elevation of the angle-tracking devices. Let i, j, k be unit vectors along the vehicle axes. Then

$$L_i = ia_{i1} + ja_{i2} + ka_{i3} \qquad (i=1,2) \tag{1}$$

Received June 8, 1977.

Index categories: Spacecraft Dynamics and Control; Spacecraft Navigation, Guidance, and Flight-Path Control.

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where

$$a_{i1} = \cos E_i \cos A_i \qquad a_{i2} = \cos E_i \sin A_i \qquad a_{i3} = \sin E_i$$

Since L_1, L_2 are not parallel, a third independent equation can be constructed from Eqs. (1) by forming the cross-product. Combining the cross-product equation with the original equations produces

$$M \begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ L_1 \times L_2 \end{bmatrix} \tag{2}$$

where

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ (a_{12}a_{23} - a_{13}a_{22}) & (a_{13}a_{21} - a_{11}a_{23}) & (a_{11}a_{22} - a_{12}a_{21}) \end{bmatrix}$$

Solving Eqs. (2)

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = M^{-1} \begin{bmatrix} L_1 \\ L_2 \\ L_1 \times L_2 \end{bmatrix} \tag{3}$$

Thus i, j, k are expressible as linear combinations of L_1, L_2, and $L_1 \times L_2$. Geometrically, sighting L_1 places the vehicle axes i, j, k somewhere on three individual cones whose common axis is L_1 and whose semivertex angles are $\cos^{-1}(a_{11})$, $\cos^{-1}(a_{12})$, $\cos^{-1}(a_{13})$, respectively. Similarly, sighting L_2 places i, j, k somewhere on three other cones whose common axis is L_2 and whose semivertex angles are $\cos^{-1}(a_{21})$, $\cos^{-1}(a_{22})$, $\cos^{-1}(a_{23})$. According to Ref. 1, there are, in general, two solutions for each of i, j, k, or eight sets of solutions altogether. The solution produced by Eq. (3) is the unique correct solution in that it enjoys the property that i, j, k is an orthogonal set satisfying the right-hand rule.

References

[1]Grubin, C., "A Simple Algorithm for Intersecting Two Conical Surfaces," *Journal of Spacecraft and Rockets*, Vol. 14, April 1977, pp. 251-252.